HETEROTIC AND TYPE I STRINGS FROM TWISTED SUPERMEMBRANES

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ABSTRACT

As shown by Hořava and Witten, there are gravitational anomalies at the boundaries of $M^{10} \times S^1/Z_2$ of 11 dimensional supergravity. They showed that only 10 dimensional vector multiplets belonging to E_8 gauge group can be consistently coupled to this theory. Thus, the dimensional reduction of this theory should be the low energy limit of the $E_8 \times E_8$ heterotic string. Here we assume that M-theory is a theory of supermembranes which includes twisted supermembranes. We show that for a target space $M^{10} \times S^1/Z_2$, in the limit in which S^1/Z_2 is small, the effective action is the $E_8 \times E_8$ heterotic string. We also consider supermembranes on $M^9 \times S^1 \times S^1/Z_2$ and find the dualities expected from 11 dimensional supergravity on this manifold. We show that the requirements for worldsheet anomaly cancellations at the boundaries of the worldvolume action are the same requirements imposed on the Hořava-Witten action.

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1 Introduction

There is strong evidence for the conjecture that all string theories can be derived from M-theory, which has as its low energy limit 11 dimensional supergavity. In [1], it was shown that the type IIA string can be obtained from a double dimensional reduction of the supermembrane whose worldvolume is a torus propagating in worldvolume time. Townsend [2] has shown that the complete spectrum of the type IIA superstring can be obtained from the supermembrane after identification of the solitonic string with the fundamental string and the solitonic 11 dimensional membrane with the fundamental membrane. In [3, 4], it was shown that 11 dimensional supergravity on K3 is dual to heterotic string on T^3. In [6], it has been shown that the strong coupling limit of type IIA superstring yields 11 dimensional supergravity. Also, the spectrum of Type IIB string on an S^1 can be identified with that of 11 dimensional supergravity on T^2 [12]. This evidence is also supplemented by other duality relations between string theories in various dimensions (see for example [6, 16, 14]).

As explained by Hořava and Witten [7], the gravitational anomaly will be non vanishing for the 11 dimensional supergravity action on $M^{10} \times S^1/Z_2$. The orbifold S^1/Z_2 has two fixed points where the manifold is singular because there is no tangent bundle defined there. Then, there will be contributions to the gravitational anomaly of the 11 dimensional supergravity action exactly at the fixed points. This anomaly is just the 10 dimensional gravitational anomaly. In order to have an anomaly free theory, there must also be additional massless states which are vector multiplets in the twisted sectors of the M-theory which can make the total anomaly, gravitational and gauge, vanish.

The gravitational anomaly in ten dimensions can be canceled by the addition of 496 vector multiplets. This means that the gauge groups which can be considered for the anomaly cancellation must have dimension 496. The possible gauge groups which arise are SO(32) and $E_8 \times E_8$. However, The gauge group $E_8 \times E_8$ is the only candidate, because there are two hyperplanes, determined by the fixed point of the orbifold [7],

each contributing symmetrically to the anomaly. Then, 248 massless vector multiplets must propagate on each hyperplane. The presence of these two hyperplanes forbids the SO(32) group because the vector multiplets can only be placed in one or the other plane.

In fact, in [8], Hořava and Witten showed that only $E_8 \times E_8$ 10 dimensional super Yang-Mills can be consistently coupled to 11 dimensional supergravity. Such a coupling requires a modified Bianchi identify for the 4-form field strength of 11 dimensional supergravity. In turn, this leads to non trivial gauge transformations of the 3-form fields of 11 dimensional supergravity.

The natural question, is how the vector multiplets are generated. To answer this question we must have some knowledge about the M-theory. As of yet we do not know which is the theory that has as its low energy limit 11 dimensional supergravity. However, there is speculation [15, 2, 4, 12] that the low energy limit of the supermembrane action is 11 dimensional supergravity. Here we make this assumption to test whether the supermembrane can reproduce the results obtained for 11 dimensional supergravity on S^1/Z_2. The motivation for such an assumption is the work of Duff et. al. [1] where it was shown that the double dimensional reduction of the supermembrane is the type IIA string.

In the next section, we review the three dimensional anomaly for the twisted solitonic membrane which should be identified with the twisted supermembrane and write the anomaly free action for the twisted supermembrane. The anomaly free conditions are shown to be the same ones as those required by the Hořava-Witten action [8]. Double dimensional reduction of this action on S^1/Z_2 is shown to yield the heterotic string with $E_8 \times E_8$ gauge group. In section 3 we consider the double dimensional reduction of the twisted supermembrane action on $S^1 \times S^1/Z_2$ to show that the dualities of 11 dimensional supergravity on $M^9 \times S^1 \times S^1/Z_2$ [7] follow from the choice of dimensional reduction. The last section is devoted to the discussions. We propose that membrane/string duality in seven dimensions [3], which is responsible

for string/string duality in six dimensions is a duality between twisted and close membranes in seven dimensions. We also suggest that the twisted membranes, which have current algebras at its boundaries, is the candidate for the construction of D-branes in M-theory.

2 Heterotic String Form The Supermembrane

As argued by Townsend [2], we must identify the solitonic membrane with the fundamental membrane of 11 dimensional supergravity in order to derive the type IIA spectrum from the supermembrane spectrum. An additional reason for such an identification is the believe that the supermembrane action has a continuous spectrum [18]. The spectrum is continuous because there is no energy needed to deform the membrane to create spikes of zero area. A way to truncate the spectrum, as suggested in [2], is to identify the supermembrane with the solitonic membrane, thus, the fundamental membrane inherits a thickness due to gravitational effects. This was shown to be the case in [19]. Due to the presence of the thickness term, spikes will necessarily have non vanishing area and will require energy to be created. This identification also allows us to obtain the properties of the supermembrane simply by studying the properties of the solitonic membrane.

In [7], the supersymmetry properties of a stable solitonic membrane on $M^{10} \times S^1/Z_2$ where studied corresponding to $x^2 = ... = x^9 = 0$ with x^1 as time , and x^{11} a coordinate on the orbifold $S^1/Z_2 \simeq I$. It was shown that the unbroken supersymmetries are given by spinors ϵ satisfying

$$\Gamma^{11}\epsilon = \epsilon$$
$$\Gamma^1\Gamma^{10}\epsilon = \epsilon. \tag{1}$$

Then, at the boundaries which are given by the fixed points of the orbifold, there will be left and right moving bosons and right moving fermions, but no left moving

fermions. We may then expect the same boundary conditions in the worldvolume of the supermembrane.

As explained in [7], the action of the solitonic membrane can have gravitational anomalies. A three dimensional action will in general not have these anomalies, but because there are boundaries, there will be effective two dimensional field theories defined at the boundaries. This in turn can give rise to gravitational anomalies much in the same manner there are 10 dimensional gravitational anomalies present in 11 dimensional supergravity theories. In addition, there is also a possibility of having gauge anomalies.

Surprisingly, we find that the worldsheet gravitational and gauge anomaly cancellation requires a modified Bianchi identity [5] which is of the same form as the modified Bianchi identity of [8] required to couple 11 dimensional supergravity to 10 dimensional super Yang-Mills. Consider a world volume given by $\mathbf{R} \times S^1 \times I$. The motivation for such a choice of worldvolume is based on the work of Hořava who constructed the type IA string from worldsheet orbifolds [9, 11]. In addition, the presence of I will allow for the double dimensional reduction leading to a heterotic string. The supermembrane action in the bulk (excluding the boundary of the worldvolume) of the theory will be [15]

$$S_M = \int d^3\zeta (\sqrt{-det(\hat{E}_i^a \hat{E}_j^b \eta_{ab})} + \frac{1}{6}\epsilon^{ijk}\hat{E}_i^A \hat{E}_j^B \hat{E}_k^C B_{CBA}). \tag{2}$$

$E_i^A = \partial_i \hat{Z}^N \hat{E}_N^A$ and $\hat{E}_M^A$ is a superelfbein and B_{CAB} is an antisymmetric 3-tensor. $Z^N = (x^n, \theta^\nu)$, $n = 1, ..., 10$ are supercoordinates on M^{10}, and Z^{11} is a supercoordinate on I. The coordinates ζ^i are worldvolume coordinates. The worldvolume part S^1 will have as coordinate σ. The worldvolume part I will have as coordinate ρ. The world volume is then a cylinder propagating in world volume time, τ, which is a coordinate on $\mathbf{R}$. The ends of the cylinder are at $\rho \pm a$.

Using the reparametrization invariances of the action S_M we set

$$\rho = x^{11}. \tag{3}$$

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The boundary will be independent of ρ. Thus, the field Z_{11} does not contribute to the gravitational anomaly because at the boundary it has no dynamics.

We must also impose the condition that the fermions which are left moving at the boundaries vanish at the boundaries in order to satisfy (1): if

$$\partial_+ \theta^{A\alpha}(\tau, \sigma, \pm a) = 0 \tag{4}$$

where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$, then

$$\theta^{A\alpha}(\tau, \sigma, \pm a) = 0. \tag{5}$$

The index α above denotes an SO(8) spacetime spinor while the index $A = 1$ labels right movers at the boundary and the index $A = 2$ labels left movers at the boundary [13]. Therefore, condition (1) implies that

$$\theta^{2\alpha}(\tau, \sigma, \pm a) = 0. \tag{6}$$

With this boundary conditions, the action for the twisted membrane takes the form

$$S_M + \int_{\partial M^3} (\sqrt{-det(E_i^a E_j^b \eta_{ab})} + \frac{1}{2}\epsilon^{ij} E_i^A E_j^B B_{BA}). \tag{7}$$

where ∂M^3 is two copies of $S^1 \times \mathbf{R}$, and the superzenbeins at the boundaries are

$$E_i^m = \partial_i X^m - i\bar{\theta}^{1\alpha} \Gamma^m \partial_i \theta^{1\alpha} \tag{8}$$

The action (2) has a worldvolume κ symmetry which insures spacetime supersymmetry in the bulk of $M^{10} \times I$. This κ symmetry has been shown to be dimensionally reduced to a worldsheet κ symmetry of the type IIA string [1], so that the κ transformations at the boundary are

$$\delta\theta^A = 2i\Gamma \cdot (\partial_i X^m - i\bar{\theta}^A \Gamma^m \partial_i \theta^A)\kappa^{Ai} \tag{9}$$

$$\delta X^m = i\bar{\theta}^A \Gamma^m \delta\theta^A. \tag{10}$$

However, the boundary condition (1) projects this κ transformations to those of action (7) which take the form

$$\delta\theta^1 = 2i\Gamma \cdot (\partial_i X^m - i\bar{\theta}^1 \Gamma^m \partial_i \theta^1)\kappa^{1i} \tag{11}$$

$$\delta X^m = i\bar{\theta}^1 \Gamma^m \delta\theta^1. \tag{12}$$

Thus, the action (7) at the boundary has the κ symmetry of the heterotic string.

Alternatively, it is possible to write the two dimensional action at the boundary in the NSR formalism in terms of Fermi superfields using (0,1) superspace which guaranties the existence of a κ symmetry required by N=1 spacetime supersymmetry [5]. This is possible because the NSR fermions are vectors of SO(8) while the fermions of the Green-Schwarz formalism are SO(8) spinors. Using the triality properties of SO(8), it is possible to map the Green-Schwarz spinors into NRS fermions transforming as vectors of SO(8). The action (7) then is the sum of two terms. One term is the Green-Schwarz supermembrane action, in addition there is an NRS heterotic string (without gauge group)

$$
\begin{aligned}
S \;=\; & S_M + \int_{\partial M^3} \{\frac{1}{2}(G_{mn}\eta^{ij} + B_{mn}\epsilon^{ij})\partial_i x^m \partial_j x^n \\
& + \frac{1}{2}i\lambda_+^a[\partial_- \lambda_+^a + \omega_m^{(+)ab}\partial_- x^m \lambda_+^b]\}
\end{aligned}
\tag{13}
$$

where G and B are the background metric and antisymmetric tensor; the latter is obtained after dimensional reduction of the 3-form tensor of the supermembrane action. η is the flat worldsheet metric and we use as in [5] the orthonormal frames E_n^a such that the right moving Majorana-Weyl fermions satisfy $\lambda^a = E_n^a \theta^n$, and we have used the triality properties of SO(8) to map the SO(8) spacetime spinor θ^ν to an SO(8) vector θ^n.

In addition, we may add an E_8 current algebra at each boundary. As shown in [8], the 10 dimensional Yang-Mills supermultiplet at each boundary which couples to 11 dimensional supergravity on a manifold $M^{10} \times I$ must be in the adjoint of the E_8 gauge group. This means that after using (3), the action (7) requires the addition of an E_8 current algebra at each boundary. Thus the total action we will consider is a sum of two terms: a bulk term and a boundary term. The bulk term will be the usual supermembrane action of [15]; the boundary term will be the non linear sigma model with (0,1) supersymmetry and E_8 gauge group. It will have a conformal anomaly, however, the bulk theory is already not conformally invariant. The explicit

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form of the boundary term can be obtained from the action considered in [5] after dropping half of the left moving fermions so that the remaining left moving fermions only couple to one E_8

$$
\begin{aligned}
S &= S_M + \int_{\partial M^3} \{ \frac{1}{2}(G_{mn}\eta^{ij} + B_{mn}\epsilon^{ij})\partial_i x^m \partial_j x^n \\
&\quad + \frac{1}{2}i\lambda_+^a [\partial_-\lambda_+^a + \omega_m^{(+)ab}\partial_- x^m \lambda_+^b] \\
&\quad + \frac{1}{2}i\psi_-^A [\partial_+\psi_+^A + A_n^{AB}\partial_+ x^n \psi_-^B] \\
&\quad + \frac{1}{4}F_{abAB}\lambda_+^a \lambda_+^b \psi_-^A \psi_-^B \},
\end{aligned}
\tag{14}
$$

The left moving Majorana-Weyl fermions ψ^A are also in the NRS formalism and only live on the boundaries; they couple naturally to A^{AB} and F_{abAB}, the Yang-Mills connections and field strength respectively, where the indices A and B run over the representation of one E_8 only. Recall that the boundary ∂M^3 is two copies of $R \times S^1$.

Thus, the closed supermembrane couples naturally to the 11 dimensional supergravity multiplet, and we expect that the twisted membrane (14) should similarly couple to the bulk supergravity with E_8 Yang-Mills super multiplets propagating at the spacetime boundary.

Action (14) will have gravitational and gauge anomalies given by the two dimensional theory. Fortunately, these anomalies have been computed for non-linear sigma models with (0,1) superspace [5]. Consider the total contribution of the anomaly to be formally described by

$$
A(R) + B(R, F_1) + B(R, F_2)
\tag{15}
$$

where $A(R)$ is the contribution from general coordinate invariance (comming from the right moving fermions at the boundary) and $B(R, F)$ is the contribution to the gauge anomaly (comming from the left moving fermions). Motivated by [7, 8] we introduce the quantity

$$
\frac{1}{2}A(R) + B(R, F)
\tag{16}
$$

at each boundary, so that (15) is the sum of the two boundary contributions (16). Therefore, in computing the gravitational and gauge anomalies, each boundary will contribute half of its usual gravitational anomaly and will contribute its full amount to the gauge anomaly comming from an E_8 multiplet.

The derivation of the anomalies have been computed in [5], where it was found that the general coordinate and gauge anomalies are canceled if the general coordinate and gauge transformations also assign a transformation of the antisymmetric two-tensor, $B_{BA} = B_{BAx^{11}}$ obtained by dimensional reduction of the the 3-form B_{BAC}

$$\delta B_{mn} = \frac{1}{2}\alpha'\{\epsilon^{AB}\partial_{[n}A_{m]}^{AB} - \frac{1}{2}\Theta^{ab}\partial_{[n}\omega_{m]}^{(-)ab}\} \tag{17}$$

and modify the Bianchi identity for the torsion $H = dB$

$$\partial_{[m}H_{npq]} = \frac{3}{16}\alpha'\{F_{[mn}^{AB}F_{pq]}^{AB} - \frac{1}{2}R_{[mn}^{(-)ab}R_{pq]}^{(-)ab}\} \tag{18}$$

where ϵ and Θ are the generators of the gauge and Lorentz transformations [2].

So the world sheet anomaly cancellation leads to the same modified Bianchi identity for H and the same transformation for the two-tensor as those obtained by requiring spacetime supersymmetry and the absence of anomalies of 11 dimensional supergravity coupled to 10 dimensional super Yang-Mills [8].

We should stress however, that the modification of the Bianchi identity (18) follows from the preservation of the κ symmetry, or alternatively of (0,1) worldsheet supersymmetry, in close analogy with [8]. As pointed out in [17], the addition of local counterterms to the effective action in order to restore gauge and Lorentz invariance breaks (0,1) supersymmetry and therefore breaks κ symmetry. Fortunately, this term can be supersymmetrized. Alternatively [5], one can work in (0,1) superspace to show that while preserving κ symmetry one is able to restore gauge and Lorentz invariance at the quantum level by using (18).

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[2] An anomaly in the general coordinate invariance is equivalent to anomaly in the Lorentz invariance [5].

The double dimensional reduction of the twisted supermembrane (14) then follows the standard procedure of [1]. We use the reparametrization invariance to set

$$x^{11} = \rho. \tag{19}$$

We then require that

$$\partial_\rho Z^N = 0 \ \ N = 1, ..., 10 \tag{20}$$

and that

$$\partial_{x^{11}} \hat{G}_{MN} = \partial_{x^{11}} \hat{B}_{MNP} = 0. \tag{21}$$

We can now express the eleven dimensional variables in term of the ten dimensional ones [1]

$$\hat{E}_M^A = \begin{pmatrix} E_M^a & E_M^\alpha + A_M \chi^\alpha & \Phi A_M \\ 0 & \chi^\alpha & \Phi \end{pmatrix}, \tag{22}$$

and

$$\hat{B}_{MNP} = (B_{MNP}, B_{MNx^{11}} = B_{MN}). \tag{23}$$

The superzenbein E^A are right moving supersymmetric and have no left moving fermions.

When I is very small, the worldvolume is pure boundary. The effective action we arrive to in this limit is given by the boundary terms in action (14),

$$
\begin{aligned}
S_h &= \int d^2\sigma \{ \frac{1}{2} (g_{mn}\eta^{ij} + b_{mn}\epsilon^{ij}) \partial_i x^m \partial_j x^n \\
&\quad + \frac{1}{2} i \lambda_+^a [\partial_- \lambda_+^a + \omega_m^{(+)ab} \partial_- x^m \lambda_+^b] \\
&\quad + \frac{1}{2} i \psi_-^A [\partial + \psi_+^A + A_n^{AB} \partial_+ x^n \psi_-^B] \\
&\quad + \frac{1}{4} F_{abAB} \lambda_+^a \lambda_+^b \psi_-^A \psi_-^B \},
\end{aligned}
\tag{24}
$$

and the gauge group indices A and B now run over the $E_8 \times E_8$ representation, rather than the E_8 representation. The action (24) is the usual heterotic string action

with $E_8 \times E_8$ gauge group and arbitrary background. As expected, the theory also has a conformal invariance which is also preserved at a quantum level. Thus, after double dimensional reduction of the non anomalous twisted supermembrane action on $M^{10} \times I$, we obtain the $E_8 \times E_8$ heterotic string. This coincides with the expected low energy limit of M-theory [7]: the 11 dimensional supergravity action on $M^{10} \times I$ should yield upon dimensional reduction the low energy theory for the heterotic string on M^{10}.

3   Heterotic-Type I duality

We now consider the action S on $M^9 \times S^1 \times I$: $E_i^A = \partial_i \hat{Z}^N \hat{E}_N^A$ and $\hat{E}_M^A$ is a superelfbein. $Z^N = (x^n, \theta^\nu)$, $N = 1, ..., 9$ are supercoordinates on M^9, Z^{10} is a coordinate on I, and Z^{11} is a supercoordinate on S^1.

The double dimensional reduction of the supermembrane then follows the same standard procedure of [1]. We use the reparametrization invariance to set

$$
\begin{aligned}
x^{10} &= \sigma \\
x^{11} &= \rho
\end{aligned}
\tag{25}
$$

and require that

$$
\partial_\rho Z^N = 0 \ \ N = 1, ..., 10
\tag{26}
$$

and that

$$
\partial_{x^{11}} \hat{G}_{MN} = \partial_{x^{11}} \hat{B}_{MNP} = 0.
\tag{27}
$$

We can now express the eleven dimensional variables in term of the ten dimensional ones [1]

$$
\hat{E}_M^A = \begin{pmatrix} E_M^a & E_M^\alpha + A_M \chi^\alpha & \Phi A_M \\ 0 & \chi^\alpha & \Phi \end{pmatrix},
\tag{28}
$$

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and

$$\hat{B}_{MNP} = (B_{MNP}, B_{MNx^{11}} = B_{MN}).$$ (29)

Before proceeding, we turn on Wilson lines on the boundaries of the supermembrane to break each E_8 to $SO(16)$. The motivation for such breaking of the gauge group is based on the fact that type IA string will be the theory we will obtain after double dimensional reduction of the supermembrane. The type IA string will be anomaly free only for gauge group $SO(16) \times SO(16)$ [9]. Therefore we consider at each end of the membrane, gauge fields A^{BC} such that

$$A_{11}^{BC} = \delta_{BC}, \quad C = 9, ..., 16,$$
$$A_m^{BC} = 0 \text{ otherwise}$$ (30)

This means that the left moving fermions with $C = 1, ..., 8$ are all massive. Therefore, upon considering the limit in which S^1 is very small, the fermions with $C = 9, ..., 16$ do not contribute to the effective action.

Thus, the effective action we arrive to when S^1 is very small is

$$
\begin{aligned}
S &= \int d^2\zeta (\Phi\sqrt{-det(E_i^a E_j^b \eta_{ab})} - \frac{1}{2}\epsilon^{ij} E_i^A E_j^B B_{BA}) \\
&+ \int d\tau \sum_{A=1}^{8} \psi_0^A(\tau, +a)\partial_+\psi_0^A(\tau, +a) \\
&+ \int d\tau \sum_{B=1}^{8} \tilde{\psi}_0^B(\tau, -a)\partial_+\tilde{\psi}_0^B(\tau, -a).
\end{aligned}
$$ (31)

The first term in (31) is the Green-Schwarz action of the type I string with Dirichlet boundary conditions for

$$x^{10} \in I.$$ (32)

The second and third terms in (31) are similar to terms first proposed by Marcus and Sagnotti in [20] to construct open string with Chan Paton factors [3]. They will

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[3] I thank P. Hořava for pointing out this reference to me

provide only zero modes, ψ_0^A and ψ_0^B, with anticommutation relationship

$$\begin{aligned}
\{\psi_0^A, \psi_0^B\} &= 2\delta^{AB}, \\
\{\tilde{\psi}_0^A, \tilde{\psi}_0^B\} &= 2\delta^{AB}.
\end{aligned} \tag{33}$$

This Clifford algebra, has a unique irreducible representation in terms of SO(8) Dirac spinors, one at each end of the open string. The open string now carries 16 dimensional indices at each end or equivalently, it carries an SO(16) representation at each end. Therefore, action (31) is just the action of the type IA string with $SO(16) \times SO(16)$ gauge group discussed in [9, 7]. This is what is expected from 11 dimensional supergravity on $S^1 \times I \times M^9$ in the limit in which the S^1 is very small [7]. What we learn from the supermembrane analysis of 11 dimensional supergravity is that the dimensional reduction of the generators of the current algebra $E_8 \times E_8$ of the heterotic string, will yield the Chan Paton factors required by the type IA string.

As explained in [9, 11], it is possible to obtain the type I string with $SO(16) \times SO(16)$ from the type IA string. Type IA has Dirichlet boundary conditions (32). As shown in [10], under a T-duality transformation, the Dirichlet boundary conditions are replaced by Neumann boundary conditions. Thus, the type IA string is T-dual to the type I string. The type I string thus obtained still has an $SO(16) \times SO(16)$ gauge group. This means that when I vanishes, which is equivalent to a T-duality transformation of the type IA string, we recover the type I string with $SO(16) \times SO(16)$ gauge group.

Alternatively, we could have considered the dimensional reduction of the x_{10} coordinate instead of the x^{11} coordinate of the supermembrane action. This would have lead to the heterotic string action with $E_8 \times E_8$ gauge group with $x^{11} \in S^1$. We can break, once more, using Wilson lines, the gauge group $E_8 \times E_8$ to $SO(16) \times SO(16)$. In the limit in which the S^1 vanishes, which is equivalent to a T-duality transformation, new massless fields appear and the whole SO(32) gauge symmetry of the heterotic string is recovered [8, 21, 22]. Thus, we recover the same result obtained in [7] by using the supermembrane. This provides more evidence for the conjecture that

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the supermembrane has as its low energy limit 11 dimensional supergravity. Moreover, the type of string we obtain is given by the choice of coordinates we decide to dimensionally reduce. This means that the underlying symmetry which is responsible for string-string duality is the reparametrization invariance of action (14). Using this symmetry, we can make the membrane look like a thin cylinder, obtaining the open string, or we can make the membrane look like a short cylinder, obtaining the heterotic string.

4 Discussion

This construction is an example where the fixed points of a worldvolume manifold provide the vector multiplets which appear in the spectrum of the low energy effective action which is believed to be 11 dimensional supergravity on an orbifold. This also provides evidence for the conjecture made in [7] that the correct number of vector multiplets appear in the spectrum needed to cancel the gravitational anomaly of 11 dimensional supergravity. We summarize the supermembrane mechanism for generating low energy effective actions with gauge group $SO(16) \times SO(16)$. Cylindrical membranes, instead of toroidal membranes, which have boundaries require an E_8 current algebra at the boundaries to cancel the 3 dimensional worldvolume anomaly present at its boundaries. This current algebras supply the $E_8 \times E_8$ current algebra of the heterotic string when the cylinder collapses to a circle. When the cylinder collapses to an orbifolded circle, the current algebra present at the boundaries are the Chan Paton factors of the open string. In order not to have an anomalous type IA theory, the E_8 current algebra must break down to $SO(16)$ at each end [7].

Perhaps the most surprising aspect of twisted supermembranes, is the fact that the spacetime anomaly cancellation and supersymmetry requirements for 11 dimensional supergravity coupled to 10 dimensional E_8 super Yang-Mills are the same requirements needed to cancel the gravitational anomaly of the twisted supermembrane.

This is more evidence supporting the idea that M-theory is a theory of closed and twisted membranes.

The perspective of M-theory being a theory of membranes, along with the fact that twisted membranes lead to heterotic strings and close membranes lead to type IIA string, suggests that membrane/string duality in seven dimensions [3], or alternatively, string/string duality in six dimensions, is actually a duality between twisted and close membranes.

Another speculation is that the D-branes of M-theory can be constructed from the twisted membranes. This follows from the fact that dimensional reduction of the twisted membrane leads to an open string with Chan Paton factors at its ends, which are present due to the algebra of the twisted membranes. With this current algebra, one can expect to construct low energy effective actions for the twisted membranes of M-theory which have gauge fields, an essential ingredient in the theory of D-branes.

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